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of dx , or Δx , is a part of BE having two ends and is, therefore, *finite*.

If the successive values of n are the numbers 1, 2, 3, 4, 5, &c., of the counting series, they are finite numbers inasmuch as the units of each of these numbers would constitute a series having a first and a last term.

Remark 4. Ci is the geometrical equivalent of $\frac{dx^2}{n^2}$, or Δx^2 ; and $n \times Ci$ or CO is the geometrical equivalent of $\frac{dx^2}{n}$, or $n \times \Delta x^2$.

As n increases without limit Ci and CO diminish without limit.

The rectangles Ci and CO have finite areas inasmuch as their sides are finite lines. Diminishing their value neither annihilates them nor changes their character as finite quantities.

Remark 5. Let $x=12$ inches, $dx=1$ inch and $n=10$.

Then will $\Delta u' = 24 \times .1 + .1^2 = 2.41$, and $n \times \Delta u' = 10 \times 2.41 = 24.1$.

If $n=100$: We have $\Delta u' = 24 \times .01 + .01^2 = .2401$, and $n \times \Delta u' = 100 \times .2401 = 24.01$.

If $n=1000$: Then $\Delta u' = 24 \times .001 + .001^2 = .024001$, and $n \times \Delta u' = 1000 \times .024001 = 24.001$.

The variable product $n \times \Delta u'$ whose successive values are 24.1, 24.01, 24.001, &c., approaches the limit 24 which is the rate of increase of the area when the side is 12 inches and its rate 1 inch per second. du' is the limit of $n \times \Delta u'$ and its value is 24.

The numerical values attributed to n and Δx are, likewise, *finite*.

Remark 6. Equation (7) is derived from equation (5) in accordance with Euclid's axiom, 3,—If equals be taken from equals, the remainders are equal.

Berkeley, Carnot, and Compte refer to a compensation of errors in the Calculus.

This compensation is really accounted for by Euclid's axiom 3, since the difference between $n \times \Delta u'$ and du' is equal to $\frac{dx^2}{n}$ the difference between the second member of (5) and $2x'dx$.

NOTE ON THE CENTROID OF PLANE AREAS.

By Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let it be required to find the centroid of the area of the curve $\frac{1}{r^m} = \frac{1}{a^m} \cos \frac{\theta}{m}$, when the density varies as the n th power of the distance of any point from the origin.

From works on Statics we get

$$\bar{x} = \frac{\int_a^\beta \int_0^r k \rho r_1^2 \cos \theta dr_1 d\theta}{\int_a^\beta \int_0^r k \rho r_1 dr_1 d\theta}$$

where ρ is the density and k the thickness of the lamina at any element.

In this case $\rho = \mu r_1^n$, and k is constant. The curve is also symmetrical to the prime vector; therefore, the abscissa being the same for the whole curve as for the half above the axis,

$$\begin{aligned} \bar{x} &= \left[\int_0^{\frac{m\pi}{2}} \int_0^r r_1^{n+2} \cos \theta dr_1 d\theta \right] / \left[\int_0^{\frac{m\pi}{2}} \int_0^r r_1^{n+1} dr_1 d\theta \right] \\ &= \frac{n+2}{n+3} \left[\int_0^{\frac{m\pi}{2}} r^{n+3} \cos \theta d\theta \right] / \left[\int_0^{\frac{m\pi}{2}} r^{n+2} d\theta \right]. \end{aligned}$$

But $r = a \cos m \frac{\theta}{m}$. Let $\frac{\theta}{m} = \varphi$, or $\theta = m\varphi$, then we get

$$\bar{x} = \frac{n+2}{n+3} a \left[\int_0^{\frac{1}{2}\pi} \cos^{mn+3m} \varphi \cos m\varphi d\varphi \right] / \left[\int_0^{\frac{1}{2}\pi} \cos^{mn+2m} \varphi d\varphi \right].$$

Now when m is an odd integer, and positive, $\cos m\varphi = A_1 \cos \varphi + A_2 \cos^3 \varphi + A_3 \cos^5 \varphi + A_4 \cos^7 \varphi + A_5 \cos^9 \varphi + A_6 \cos^{11} \varphi + A_7 \cos^{13} \varphi + A_8 \cos^{15} \varphi + A_9 \cos^{17} \varphi + A_{10} \cos^{19} \varphi + \dots$,

where $A_1 = \frac{m}{(-1)^{\frac{m-1}{2}}}$, $A_2 = -\frac{m(m^2-1)}{(-1)^{\frac{m-1}{2}} |3|}$, $A_3 = \frac{m(m^2-1)(m^2-3^2)}{(-1)^{\frac{m-1}{2}} |5|}$, &c.

$$\begin{aligned} \therefore \bar{x} &= \frac{n+2}{n+3} a \left[\int_0^{\frac{1}{2}\pi} \left(A_1 \cos^{mn+3m+1} \varphi + A_2 \cos^{mn+3m+3} \varphi + A_3 \cos^{mn+3m+5} \varphi \right. \right. \\ &\quad + A_4 \cos^{mn+3m+7} \varphi + A_5 \cos^{mn+3m+9} \varphi + A_6 \cos^{mn+3m+11} \varphi \\ &\quad + A_7 \cos^{mn+3m+13} \varphi + A_8 \cos^{mn+3m+15} \varphi + A_9 \cos^{mn+3m+17} \varphi \\ &\quad \left. \left. + A_{10} \cos^{mn+3m+19} \varphi + \dots \right) d\varphi \right] / \left[\int_0^{\frac{1}{2}\pi} \cos^{mn+2m} \varphi d\varphi \right]. \\ \bar{x} &= \frac{n+2}{n+3} a \left[A_1 \frac{\Gamma\left(\frac{mn+3m+2}{2}\right)}{\Gamma\left(\frac{mn+3m+3}{2}\right)} + A_2 \frac{\Gamma\left(\frac{mn+3m+4}{2}\right)}{\Gamma\left(\frac{mn+3m+5}{2}\right)} + A_3 \frac{\Gamma\left(\frac{mn+3m+6}{2}\right)}{\Gamma\left(\frac{mn+3m+7}{2}\right)} \right. \\ &\quad \left. + A_4 \frac{\Gamma\left(\frac{mn+3m+8}{2}\right)}{\Gamma\left(\frac{mn+3m+9}{2}\right)} + A_5 \frac{\Gamma\left(\frac{mn+3m+10}{2}\right)}{\Gamma\left(\frac{mn+3m+11}{2}\right)} + A_6 \frac{\Gamma\left(\frac{mn+3m+12}{2}\right)}{\Gamma\left(\frac{mn+3m+13}{2}\right)} \right. \\ &\quad \left. + A_7 \frac{\Gamma\left(\frac{mn+3m+14}{2}\right)}{\Gamma\left(\frac{mn+3m+15}{2}\right)} + A_8 \frac{\Gamma\left(\frac{mn+3m+16}{2}\right)}{\Gamma\left(\frac{mn+3m+17}{2}\right)} + A_9 \frac{\Gamma\left(\frac{mn+3m+18}{2}\right)}{\Gamma\left(\frac{mn+3m+19}{2}\right)} \right. \\ &\quad \left. + A_{10} \frac{\Gamma\left(\frac{mn+3m+20}{2}\right)}{\Gamma\left(\frac{mn+3m+21}{2}\right)} + \dots \right] \end{aligned}$$

$$\begin{aligned}
& + A_7 \frac{\Gamma\left(\frac{mn+3m+14}{2}\right)}{\Gamma\left(\frac{mn+3m+15}{2}\right)} + A_8 \frac{\Gamma\left(\frac{mn+3m+16}{2}\right)}{\Gamma\left(\frac{mn+3m+17}{2}\right)} + A_9 \frac{\Gamma\left(\frac{mn+3m+18}{2}\right)}{\Gamma\left(\frac{mn+3m+19}{2}\right)} \\
& + A_{10} \frac{\Gamma\left(\frac{mn+3m+20}{2}\right)}{\Gamma\left(\frac{mn+3m+21}{2}\right)} + \dots \Bigg] \Bigg/ \left[\frac{\Gamma\left(\frac{mn+2m+1}{2}\right)}{\Gamma\left(\frac{mn+2m+2}{2}\right)} \right].
\end{aligned}$$

Let $m=1$, then

$$\bar{x} = \frac{n+2}{n+3} a \left[\frac{\Gamma\left(\frac{n+5}{2}\right)}{\Gamma\left(\frac{n+6}{2}\right)} \right] \Bigg/ \left[\frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{n+4}{2}\right)} \right] = \frac{n+2}{n+3} \left[\frac{n+3}{n+4} \right] a = \frac{n+2}{n+4} a.$$

When $n=0$, $\bar{x} = \frac{1}{2} a$ $\therefore$ the centre of the circle is its centroid.

Let $m=3$, then

$$\bar{x} = \frac{n+2}{n+3} a \left[4 \frac{\Gamma\left(\frac{3n+13}{2}\right)}{\Gamma\left(\frac{3n+14}{2}\right)} - 3 \frac{\Gamma\left(\frac{3n+11}{2}\right)}{\Gamma\left(\frac{3n+12}{2}\right)} \right] \Bigg/ \left[\frac{\Gamma\left(\frac{3n+7}{2}\right)}{\Gamma\left(\frac{3n+8}{2}\right)} \right]$$

$$\begin{aligned}
\bar{x} &= \frac{n+2}{n+3} \left[4 \frac{(3n+11)(3n+9)(3n+7)}{(3n+12)(3n+10)(3n+8)} - 3 \frac{(3n+9)(3n+7)}{(3n+10)(3n+8)} \right] a \\
&= \frac{n+2}{n+3} \left[\frac{(3n+9)(3n+7)}{(3n+12)(3n+10)} \right] a = \frac{(n+2)(3n+7)}{(n+4)(3n+10)} a.
\end{aligned}$$

Let $m=5$, then

$$\bar{x} = \frac{n+2}{n+3} a \left[5 \frac{\Gamma\left(\frac{5n+17}{2}\right)}{\Gamma\left(\frac{5n+18}{2}\right)} - 20 \frac{\Gamma\left(\frac{5n+19}{2}\right)}{\Gamma\left(\frac{5n+20}{2}\right)} + 16 \frac{\Gamma\left(\frac{5n+21}{2}\right)}{\Gamma\left(\frac{5n+22}{2}\right)} \right] \Bigg/ \left[\frac{\Gamma\left(\frac{5n+11}{2}\right)}{\Gamma\left(\frac{5n+12}{2}\right)} \right]$$

$$\begin{aligned}
\bar{x} &= \frac{n+2}{n+3} \left[5 \frac{(5n+15)(5n+13)(5n+11)}{(5n+16)(5n+14)(5n+12)} - 20 \frac{(5n+17)(5n+15)(5n+13)(5n+11)}{(5n+18)(5n+16)(5n+14)(5n+12)} \right. \\
&\quad \left. + 16 \frac{(5n+19)(5n+17)(5n+15)(5n+13)(5n+11)}{(5n+20)(5n+18)(5n+16)(5n+14)(5n+12)} \right] a \\
&= \frac{n+2}{n+3} \left[\frac{(5n+15)(5n+13)(5n+11)}{(5n+20)(5n+18)(5n+16)} \right] a = \frac{(n+2)(5n+11)(5n+13)}{(n+4)(5n+16)(5n+18)} a.
\end{aligned}$$

The rule is now established. The first factor is always $\frac{n+2}{n+4}$, the numerator of the second factor is $mn+2m+1$ the denominator is $mn+3m+1$, the numerator of each succeeding factor is 2 greater than the one immediately preceding, so also for each succeeding factor in the denominator.

Let $m=7$, then $mn+2m+1=7n+15$, and $mn+3m+1=7n+22$ and

$$\bar{x} = \frac{(n+2)(7n+15)(7n+17)(7n+19)}{(n+4)(7n+22)(7n+24)(7n+26)} a$$

The number of factors when $m=1$, is 1, for $m=3$, is 2, for $m=5$, is 3 for $m=7$, is 4 and so on,

Let $m=9$, then $mn+2m+1=9n+19$; $mn+3m+1=9n+28$.

$\therefore \bar{x} = \frac{(n+2)(9n+19)(9n+21)(9n+23)(9n+25)}{(n+4)(9n+28)(9n+30)(9n+32)(9n+34)} a$, and so on for any value of m .

The centroid is evidently on the axis of symmetry. $\therefore \bar{x}=0$.

When $m=9$, $\bar{x}$ reduces to the following

$$\bar{x} = \frac{(n+2)(3n+7)(9n+19)(9n+23)(9n+25)}{(n+4)(3n+10)(9n+28)(9n+32)(9n+34)} a.$$

When m is an even interger, and positive,

$$\cos m \varphi = A_1 + A_2 \cos^2 \varphi + A_3 \cos^4 \varphi + A_4 \cos^6 \varphi + A_5 \cos^8 \varphi + A_6 \cos^{10} \varphi + A_7 \cos^{12} \varphi + A_8 \cos^{14} \varphi + A_9 \cos^{16} \varphi + A_{10} \cos^{18} \varphi + \dots,$$

$$\text{where } A_1 = \frac{1}{(-1)^{\frac{m}{2}}}, A_2 = -\frac{m^2}{(-1)^{\frac{m}{2}} 2}, A_3 = \frac{m^2(m^2-2^2)}{(-1)^{\frac{m}{2}} \frac{4}{2}} \&c.$$

$$\begin{aligned} \bar{x} = \frac{n+2}{n+3} a \left[\int_0^{i\pi} \left(A_1 \cos^{mn+3m} \varphi + A_2 \cos^{mn+3m+2} \varphi + A_3 \cos^{mn+3m+4} \varphi \right. \right. \\ \left. \left. + A_4 \cos^{mn+3m+6} \varphi + A_5 \cos^{mn+3m+8} \varphi + A_6 \cos^{mn+3m+10} \varphi + A_7 \cos^{mn+3m+12} \varphi \right. \right. \\ \left. \left. + A_8 \cos^{mn+3m+14} \varphi + A_9 \cos^{mn+3m+16} \varphi + \right. \right. \end{aligned}$$

$$\left. A_{10} \cos^{mn+3m+18} \varphi + \dots \right) d\varphi \Bigg] \Bigg/ \left[\int_0^{i\pi} \cos^{mn+2m} \varphi d\varphi \right].$$

$$\bar{x} = \frac{n+2}{n+3} a \left[A_1 \frac{\Gamma\left(\frac{mn+3m+1}{2}\right)}{\Gamma\left(\frac{mn+3m+2}{2}\right)} + A_2 \frac{\Gamma\left(\frac{mn+3m+3}{2}\right)}{\Gamma\left(\frac{mn+3m+4}{2}\right)} + A_3 \frac{\Gamma\left(\frac{mn+3m+5}{2}\right)}{\Gamma\left(\frac{mn+3m+6}{2}\right)} \right]$$

$$\begin{aligned}
& + A_4 \frac{\Gamma\left(\frac{mn+3m+7}{2}\right)}{\Gamma\left(\frac{mn+3m+8}{2}\right)} + A_5 \frac{\Gamma\left(\frac{mn+3m+9}{2}\right)}{\Gamma\left(\frac{mn+3m+10}{2}\right)} + A_6 \frac{\Gamma\left(\frac{mn+3m+11}{2}\right)}{\Gamma\left(\frac{mn+3m+12}{2}\right)} \\
& + A_7 \frac{\Gamma\left(\frac{mn+3m+13}{2}\right)}{\Gamma\left(\frac{mn+3m+14}{2}\right)} + A_8 \frac{\Gamma\left(\frac{mn+3m+15}{2}\right)}{\Gamma\left(\frac{mn+3m+16}{2}\right)} + A_9 \frac{\Gamma\left(\frac{mn+3m+17}{2}\right)}{\Gamma\left(\frac{mn+3m+18}{2}\right)} \\
& + A_{10} \frac{\Gamma\left(\frac{mn+3m+19}{2}\right)}{\Gamma\left(\frac{mn+3m+20}{2}\right)} + \dots \Bigg] \Bigg/ \left[\frac{\Gamma\left(\frac{mn+2m+1}{2}\right)}{\Gamma\left(\frac{mn+2m+2}{2}\right)} \right]
\end{aligned}$$

Let $m=2$, then

$$\begin{aligned}
\bar{x} &= \frac{n+2}{n+3} a \left[2 \frac{\Gamma\left(\frac{2n+9}{2}\right)}{\Gamma\left(\frac{2n+10}{2}\right)} - \frac{\Gamma\left(\frac{2n+7}{2}\right)}{\Gamma\left(\frac{2n+8}{2}\right)} \right] \Bigg/ \left[\frac{\Gamma\left(\frac{2n+5}{2}\right)}{\Gamma\left(\frac{2n+6}{2}\right)} \right] \\
\bar{x} &= \frac{n+2}{n+3} \left[2 \frac{(2n+7)(2n+5)}{(2n+8)(2n+6)} - \frac{2n+5}{2n+6} \right] a = \frac{(n+2)(2n+5)}{(n+3)(2n+8)} a.
\end{aligned}$$

Let $m=4$, then

$$\begin{aligned}
\bar{x} &= \frac{n+2}{n+3} a \left[\frac{\Gamma\left(\frac{4n+13}{2}\right)}{\Gamma\left(\frac{4n+14}{2}\right)} - 8 \frac{\Gamma\left(\frac{4n+15}{2}\right)}{\Gamma\left(\frac{4n+16}{2}\right)} + 8 \frac{\Gamma\left(\frac{4n+17}{2}\right)}{\Gamma\left(\frac{4n+18}{2}\right)} \right] \Bigg/ \left[\frac{\Gamma\left(\frac{4n+9}{2}\right)}{\Gamma\left(\frac{4n+10}{2}\right)} \right] \\
\bar{x} &= \frac{n+2}{n+3} \left[\frac{(4n+11)(4n+9)}{(4n+12)(4n+10)} - 8 \frac{(4n+13)(4n+11)(4n+9)}{(4n+14)(4n+12)(4n+10)} \right. \\
& \quad \left. + 8 \frac{(4n+15)(4n+13)(4n+11)(4n+9)}{(4n+16)(4n+14)(4n+12)(4n+10)} \right] a, \quad \bar{x} = \frac{(n+2)(4n+9)(4n+11)}{(n+3)(4n+14)(4n+16)} a.
\end{aligned}$$

The first factor is always $\frac{n+2}{n+3}$, the numerator of the second factor is $mn+2m+1$, the denominator $mn+3m+2$, the numerator of each succeeding factor is 2 greater than the one immediately preceding, so also for each succeeding factor in the denominator.

The number of factors for $m=2$, is 2, for $m=4$, is 3, for $m=6$, is 4 and so on.

Let $m=6$, then $mn+2m+1=6n+13$, $mn+3m+2=6n+20$ and

$$\bar{x} = \frac{(n+2)(6n+13)(6n+15)(6n+17)}{(n+3)(6n+20)(6n+22)(6n+24)} a = \frac{(n+2)(2n+5)(6n+13)(6n+17)}{(n+3)(2n+8)(6n+20)(6n+22)} a.$$

Let $m=8$, then $mn+2m+1=8n+17$, $mn+3m+2=8n+26$ and

$$\bar{x} = \frac{(n+2)(8n+17)(8n+19)(8n+21)(8n+23)}{(n+3)(8n+26)(8n+28)(8n+30)(8n+32)} a, \text{ and so on for any value of } m.$$

The centroid is on the axis of symmetry. $\therefore \bar{y}=0$.

If m be a positive fraction the centroid will be the origin and $\bar{x}=0$, $\bar{y}=0$.

This follows from the fact that there are as many loops, (equal) arranged about the centre as the denominator of the fraction represented by m has integers in it. Hence, if $m=\frac{q}{p}$, then there are p equal loops around the centre.

A general rule is as follows:

When m is a positive odd integer

$$\bar{x} = \frac{n+2}{n+3} \left[\frac{(mn+2m+1)(mn+2m+3)(mn+2m+5) \dots}{(mn+3m+1)(mn+3m+3)(mn+3m+5) \dots} \right] a, \text{ to } \frac{m+1}{2} \text{ factors inside the brackets.}$$

When m is a positive even integer

$$\bar{x} = \frac{n+2}{n+3} \left[\frac{(mn+2m+1)(mn+2m+3)(mn+2m+5) \dots}{(mn+3m+2)(mn+3m+4)(mn+3m+6) \dots} \right] a, \text{ to } \frac{m}{2} \text{ factors inside the brackets. In either case } \bar{y}=0,$$

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

9. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburgh, Logan County, Ohio.

Four logs of uniform thickness whose diameters are each four feet, lie side by side and touch each other. In the crevices of these logs lie three logs 3 feet in diameter, and in the crevices of the three logs lie two logs whose diameters are 2 feet. What must be the diameter of a log to lie on the top of the pile and touch the two logs and the middle one of the three logs?